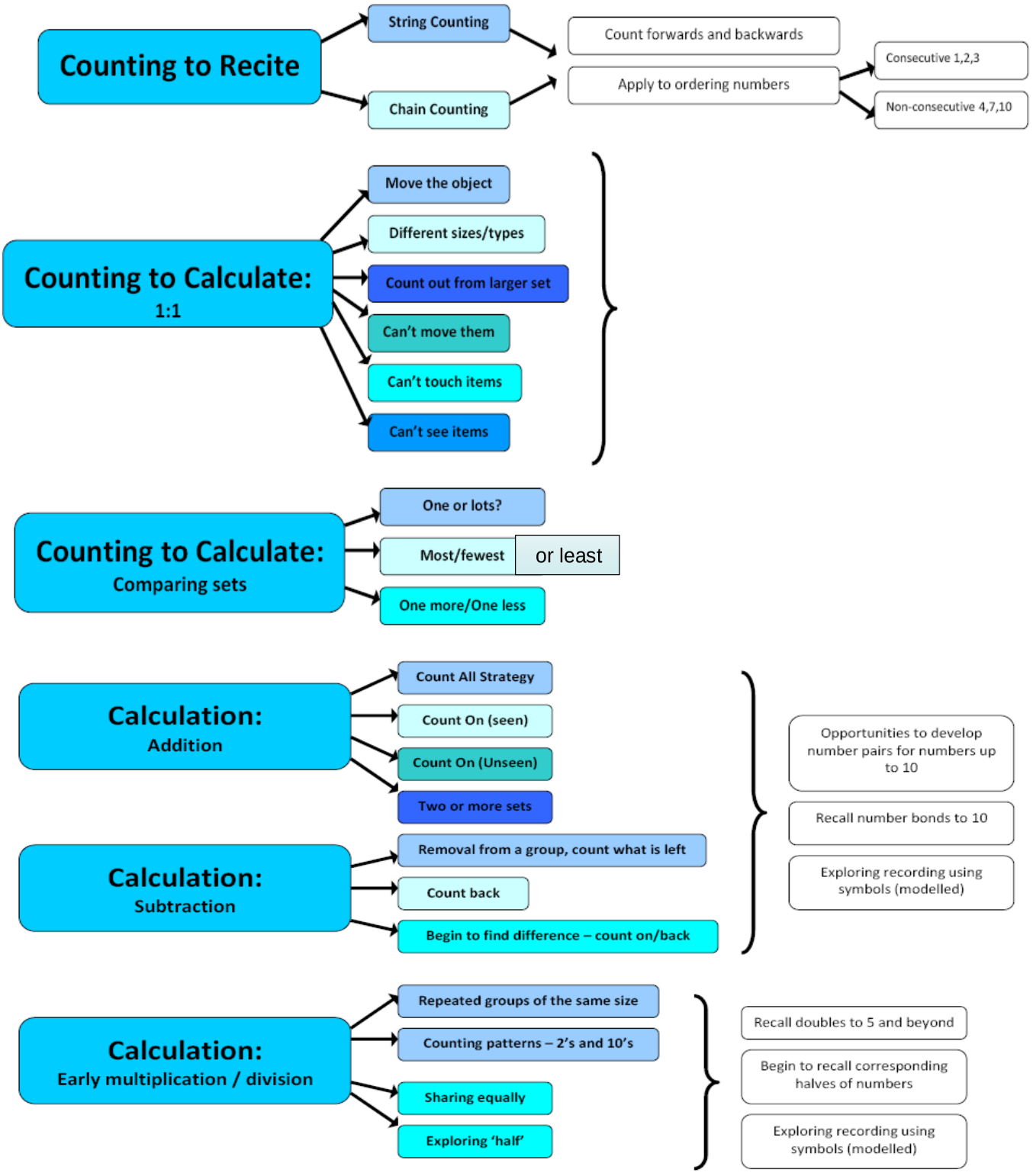
Counting into Calculating – A guide to progression



NB. Opportunities for children to meet these 5 core skills should be integrated. Children **DO NOT** have to be able to recite ALL numbers before they move to calculation. For example, if they are able to recite numbers to 3 then **they can** 1:1, compare, add and subtract up to 3.

Progression in Addition

Addition is commutative.
Addition of positive numbers will give a larger answer than the start number as you are adding to the set.

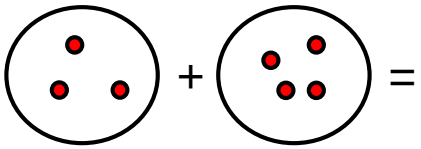
Simpler Case → Crossing boundary → '0' as a place holder → Both involved → Mixed number of digits → More than 2 sets involved

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

3 + 4 = 7

*Use of objects, **number tracks & number lines***



Y1/Y2

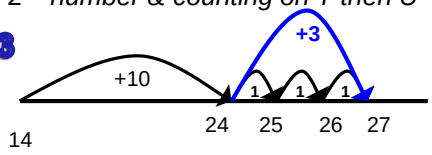
Use of 100 squares to add tens then units.

Larger number 1st

13 + 14 = 27

Number lines – secure partitioning
2nd number & counting on T then U

Y2/Y3



Y3/4 Expanded

23 + 33 =

T	U
30	+ 3
20	+ 3
50	**+ 6 = 56**

46 + 28 =

T	U
40	+ 6
20	+ 8
10	
70	**+ 4 = 74**

4.3 + 18.6 =

T	U	.	t
10	+ 8	+	0.6
	4	+	0.3
10			
20	**+ 2**	**+**	**0.9 = 22.9**

Children must be taught to add smallest digit first i.e. units/tenths etc. Children also taught to estimate first.

Link to money for understanding

Standard Algorithm

Y5/6

```
  1 2 6
+ 4 3
-------
  1 6 9
-------
```

Headings of Th, H,T,U,t, h, can be used to support children in both Y5 and Y6 if needed. Can be omitted when children feel confident.

```
  2 8
+ 4 3
-------
    7 1
-------
  1
```

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789 + 642 becomes

```
  7  8  9
+ 6  4  2
----------
1  4  3  1
   1  1
```

Answer: 1431

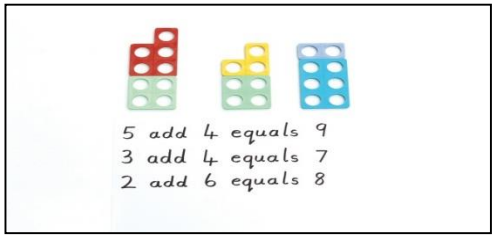
- Understanding of place value places
- Add multiples of 10 (100) fluently
- Partition numbers into HTU and recombining
- Fluent in adding single digit numbers to 20

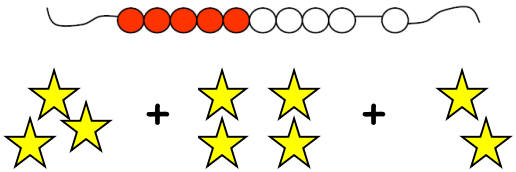
U+U TU+U TU+TU HTU+U HTU+TU HTU+HTU ThHTU U+0.t 0.t+0.t U+0.t h Mixed whole numbers & decimals

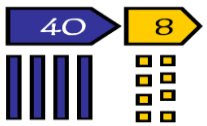
Misconceptions

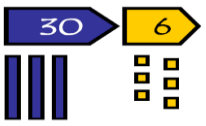
- Estimating first to see if their answer ‘makes sense’
- Setting out when working in columns – confusion over the place value
- Confusion of ‘teen’ and ‘ty’
- Using in number line – count start number so calculation is out by 1

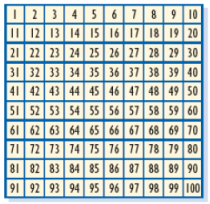
Models & Images





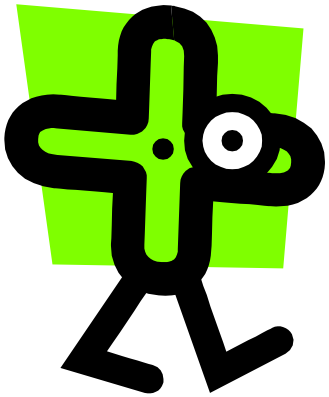






Linked Vocabulary

Add
More
Sum
Total
Make
Greater
Plus
Addition
Increase



Progression in Subtraction (removal from set, decomposition)

- *Can be removal from set or finding the difference*
- *It is NOT commutative*

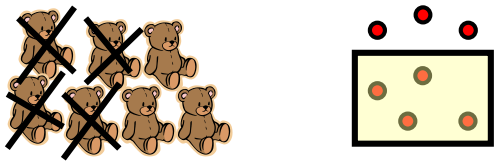
Simpler Case → Crossing boundary → '0' as a place holder → Both involved → Mixed number of digits → More than 2 sets involved

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

7 – 4 =

Use of objects, number tracks & number lines



See progression – counting to calculating

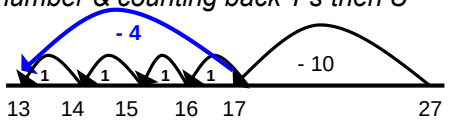
Y1/Y2

27 - 14 = 13

Use of 100 squares to take tens then units.

Y2/Y3

***Number lines** – secure partitioning 2nd number & counting back T's then U*



Y3/4 Expanded

37 - 14 =

	T	U
	30	7
-	10	4
	20	**+ 3 = 23**

46 - 28 =

30

40 + 6

- 20 + 8

10 + 8 = 18

12.6 – 4.3 =

	10	12	0.6
-		4	0.3
		8	**+ 0.3 = 8.3**

Children must be taught to take smallest digit first i.e. units/tenths etc. Children also taught to estimate first.

Compact Y4/5

3 1

4 6

- 2 8

1 8

12

1 2 6

- 4 3

8 3

Headings of Th, H,T,U,t, h, can be used to support children in both Y5 and Y6 if needed. Can be omitted when children feel confident.

Standard Algorithm Y5/6

3 1

4 6

- 2 8

1 8

3 1

4 2 . 9

- 1 3 . 2

2 9 . 7

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932 – 457 becomes

8 12 1

9 3 2

– 4 5 7

4 7 5

- Subtract single digits confidently mentally
- Recall number bonds up to 20
- Partition numbers and recombine
- Understand place value and 'exchange'
- Fluent in subtracting multiples of 10/100/100

U-U TU-U TU-TU HTU-U HTU-TU HTU-HTU ThHTU U-0.t 0.t-0.t U-0.t h Mixed whole numbers & decimals

Progression in Subtraction (finding the difference)

- *Can be removal from set or finding the difference*
- *Can count on or back to find the difference*
- *Removal from set is not commutative*

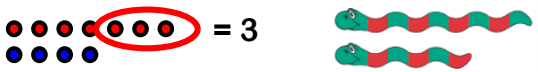
Simpler Case → Crossing boundary → '0' as a place holder → Both involved → Mixed number of digits → More than 2 sets involved

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

What is the difference between 7 and 4

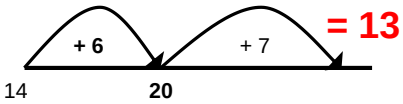
Use of objects, number tracks & number lines



27 - 14 = 13

Number lines – can count fwds/bwds, bridge to T then multiples of 10

Y1/Y2

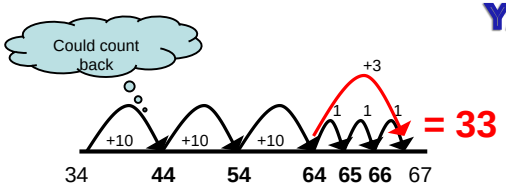


67 – 34 =

Number lines – without bridging to the nearest 10

Use of 100 squares to count on.
***Number lines** – secure partitioning 2nd number & counting on or back T's then U*

Y2/Y3

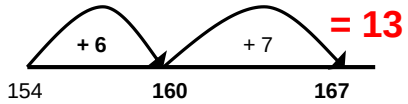


Y3/4 Expanded

Complimentary Addition: *counting up*

167 – 154 =

Counting up to the next ten and using number bonds. Then in tens, then any extra units.



Children should be taught to use counting on when the numbers are close together.

- Subtract single digits confidently mentally
- Bridge to the nearest 10
- Add multiple to 10/100 to multiples / count fwds/bwds in 10/100 from any given number
- Partition into HTU
- Add several numbers mentally
- Secure addition strategy

Compact

Y4/5

67 – 34 =

```
 6 7
- 3 4
-------
     6   (40)
 2 0     (60)
     7   (67)
-------
 3 3
```

```
  754
- 286
------
   14  (300)
  454  (754)
  468
```

Standard Algorithm

Y5/6

N/A for Complimentary addition

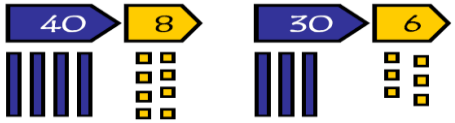
Can use a count back strategy rather than count up

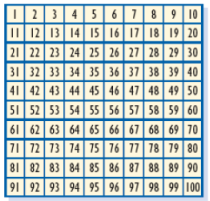
U-U TU-U TU-TU HTU-U HTU-TU HTU-HTU ThHTU U-0.t 0.t-0.t U-0.t h Mixed whole numbers & decimals

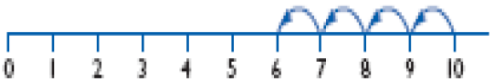
Misconceptions

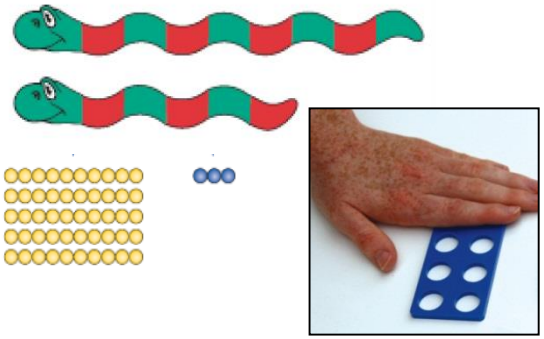
- Estimating first to see if their answer 'makes sense'
- Setting out when working in columns – confusion over the place value
- Confusion of 'teen' and 'ty'
- Using in number line – count start number so calculation is out by 1
- Misunderstanding regarding place value and concept of exchanging **T** for ones, **H** for **T**ens etc
- Lack of understanding that when subtracting from a number that the answer will be smaller than start number as removing from it
- Children switch the digits around to be able to 'do' the calculation (believe it is commutative as with +/x)

Models & Images









Linked Vocabulary

Take
Take-away
Leave
Left
Fewer
Less than
Decrease
Difference between
Minus
Subtract
Subtraction

Progression in Multiplication (Short multiplication)

Multiplication is commutative.
Answer will be larger

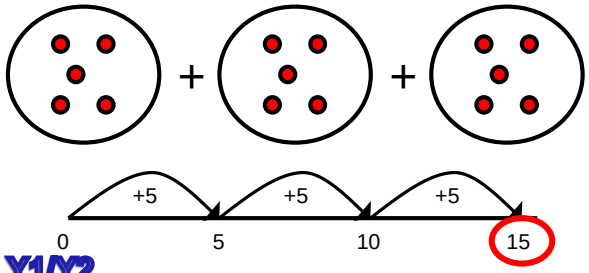
Simpler Case (1x 10/100 – as in examples given) → Multiples of 10/100 (3x235) → '0' as a place holder → Both involved → Mixed number of digits

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

3 lots of 5 = 15

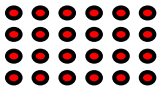
Use of objects, number tracks & number lines. Link initially to repeated +



Y1/Y2

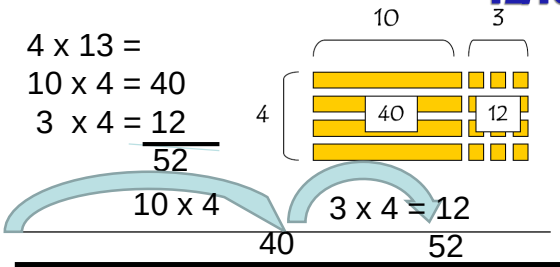
Representation as an array using a variety of apparatus (Dienes, pegs, counters etc)

4 x 6 =



Y2/Y3

4 x 13 =
10 x 4 = 40
3 x 4 = 12
52



- Recall tables up to 12x12 (by the end Y4)
- Partition numbers into HTU
- Multiply by 10/100
- Secure addition strategy for calculating total

Y3/4 Expanded

Grid Method

3 x 15 =

x	10	5
3	30	15

= 45

3 x 126 =

x	100	20	6
3	300	60	18

= 378

22 x 14 =

x	20	2
10	200	20
4	80	8

220
88
308

Compact Y4/5

15
x 3
15 (3x5)
30 (3x10)
45

126
x 3
18 (3x6)
60 (3x20)
300 (3x100)
378

NB. As children become secure in this method there may no longer be a need to write the calculations in brackets

Standard Algorithm Y5/6

15
x 3
45
1

126
x 3
378
1

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2741 × 6 becomes

	2	7	4	1
×				6
1	6	4	4	6
	4	2		

Answer: 16 446

UxU UxTU UxHTU UxThHTU Ux0.t U+0.t h Mixed whole numbers & decimals

Progression in Multiplication (Long multiplication)

Multiplication is commutative.
Answer will be larger

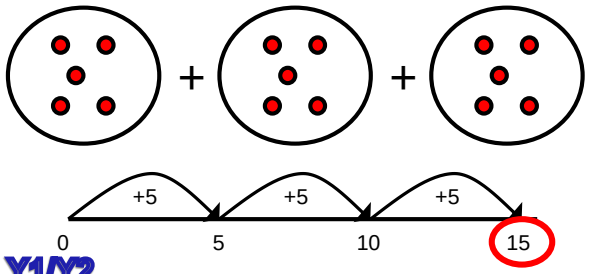
Simpler Case (1x 10/100 – as in examples given) → Multiples of 10/100 (3x235) → '0' as a place holder → Both involved → Mixed number of digits

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

3 lots of 5 = 15

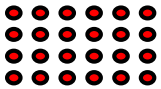
Use of objects, number tracks & number lines. Link initially to repeated +



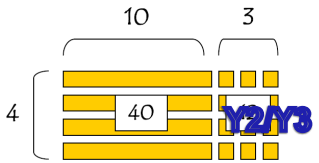
Y1/Y2

Representation as an array using a variety of apparatus (Dienes, pegs, counters etc)

4 x 6 =



4 x 13 =



Y2/Y3

**** NB. use apparatus to model TUxTU & HTUxTU etc***

Y3/4 Expanded

Grid Method

14 x 13 =

	10	3
10	100	30
4	40	12

= 182

14 x 123 =

	100	20	3
10	1000	200	30
4	400	80	12

= 1722

Compact

Y4/5

```
  1 4
x 1 3
-----
  1 2   (3x4)
  3 0   (10x3)
  4 0   (10x4)
1 0 0   (10x10)
-------
  1 8 2
```

NB. As children become secure in this method there may no longer be a need to write the calculations in brackets

```
  1 2 3
x   1 4
-------
  1 2   (4x3)
  8 0   (4x20)
4 0 0   (100x4)
  3 0   (10x3)
2 0 0   (10x20)
1 0 0 0 (100x10)
---------
1 7 2 2
  1
```

Standard Algorithm

Y5/6

```
  1 4
x 1 3
-----
  4 2
1 4 0
-----
1 8 2
```

Colours maybe used for clarity

```
  1 2 4
x   2 6
-------
  7 4 4
2 4 8 0
-------
3 2 2 4
1 1
```

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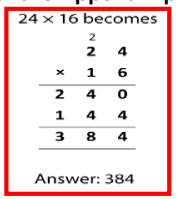
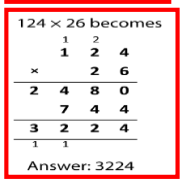




- Recall tables up to 12x12 (by the end Y4)
- Partition numbers into HTU
- Multiply by 10/100
- Secure addition strategy calculating total

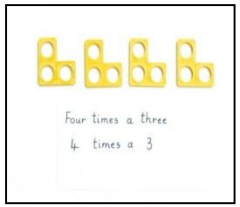
TUxTU TUxHTU ThHTUxTU Decimals up to 2dp x whole numbers

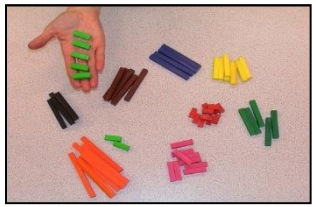
Misconceptions

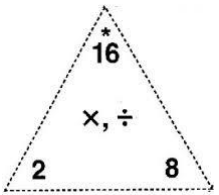
- Understanding on multiplying by 10/100 and what happens to place value of the number
- Rapid recall of multiplication tables is not secure and impacting of accuracy of calculation
- Interpretation of digits in the T/H columns as single digits eg 4x3 instead of 4x30
- Children should be taught to recall multiplication facts and given strategies to quickly work out unknown facts.

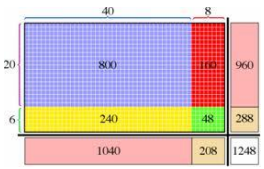
Year One – 2, 5 and 10
Year Two – 2, 5, 10 and 3
Year Three - 2, 5, 10, 3, 4 and 8
Year Four – all tables.

Models & Images









Linked Vocabulary

Repeated addition
Groups of
Lots of
Multiply
Times
Multiplication
Product
Array

Progression in Division (short division)

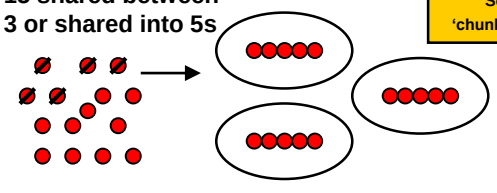
- *Division can be sharing or grouping*
- *Division is the inverse of multiplication*

Simpler Case → Within known x tables → Using multiples → Mixture of tens and ones → Within less familiar x tables

Context based experiences at each level of development - money, measures, real-life

F2 and Y1 Developmental

15 shared between 3 or shared into 5s



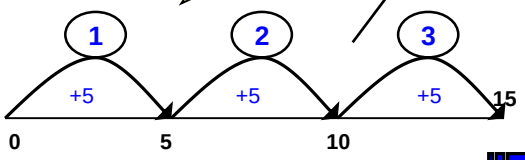
Use of objects, number tracks & number lines. Link initially to repeated subtraction. Move towards arrays in Y2

15 ÷ 5 = 3

Y1/Y2

I need to get from 0 to 15 in jumps of 5

3 groups of 5



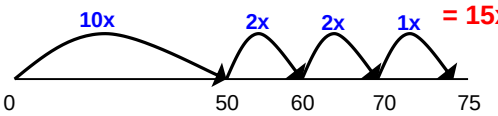
Multiplication and division symbols will be introduced in Y2.

Y3/4 Expanded

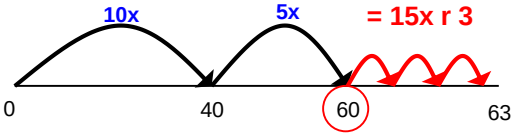
School will need to decide to 'chunk' either forwards or backwards

Children are encouraged to chunk with larger multiples of the divisor → 10x, 5x, 20x, 4x

75 ÷ 5 = 15

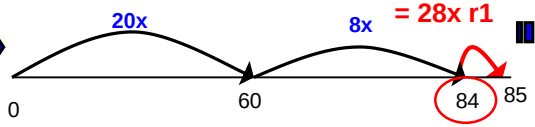


63 ÷ 4 = 15 r3



85 ÷ 3 = 28 r1

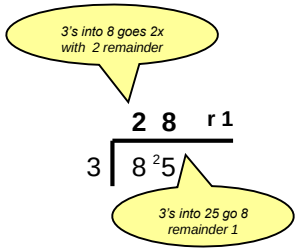
Remainders should also be recorded as a decimal or fraction

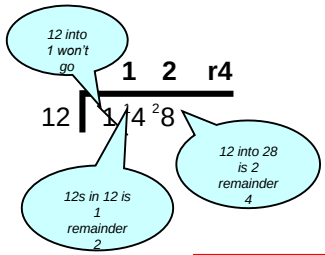


If moving to ÷ 3digit number by a single digit, then the standard algorithm should be used.

Y5/6 Standard Algorithm

'Stand alone' method → for efficiency





- Count in multiples of # from/back from 0
- Recall known facts for all multiplication tables up to 12x12 (NB multiplication squares can be used as a supporting model/image)
- Know related facts – doubling, x10
- Confidently subtract mentally

TU ÷ U → HTU ÷ U → ThHTU ÷ U → U.t ÷ U → U.t h ÷ U → TU.u ÷ U → HTU ÷ TU

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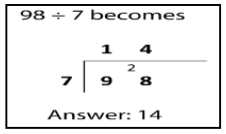
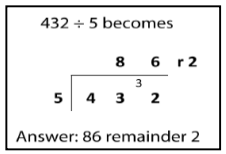




Progression in Division (long division)

- *Division can be sharing or grouping*
- *Division is the inverse of multiplication*

Simpler Case → Within known/derived x tables → Any two-digit number

Context based experiences at each level of development - money, measures, real-life

Developmental

Secure division strategies for short division, recall and application of known & derived facts and making sensible approximations to support accuracy when calculating

Y5/6 Expanded

432 ÷ 15 becomes

			2	8 r 12
1	5	4	3	2
(X20)		3	0	0
		1	3	2
(X8)		1	2	0
			1	2

Remainders should also be recorded as a decimal or fraction ie 12/15

15 into 4 won't go

2 8 r12

15 | 4 4 3 13 2

15 into 43 is 2 remainder 13

15's in 132 is ...

At each stage children may need to make jottings to support the use of known and derived facts when calculating

Compact

Y5/6

1 5 r1

23 | 3 4 6

(1 x 23) - 2 3

1 1 6

(5 x 23) - 1 1 5

0 0 1

0 2 3 r10

33 | 7 7 6 10 9

2 x 33 = 66
3 x 33 = 99
10 x 33 = 330
5 x 33 = 165

= 23r10 or 23 10/33

Standard Algorithm

Y6 – extension

2 3 r10

33 | 7 6 9

- 6 6

1 0 9

- 9 9

1 0

Conversion of remainders into fractions then decimals should be taught.

0 2 3 . 3 0 (2dp)

33 | 7 7 6 10 9 . 10 0 10

Children will need to be aware that the method for long division will not work with every calculation e.g.

24 | 2 3 7

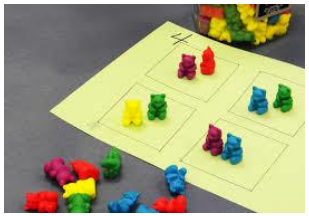
Therefore they MUST have an alternative method they can use.

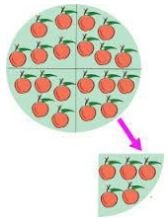
- Recall known facts for all multiplication tables up to 12x12 (NB multiplication squares can be used as a supporting model/image)
- Know related facts – doubling/halving, x10 x50 x25 x100
- Apply tables knowledge when approximating answers
- Confidently subtract mentally

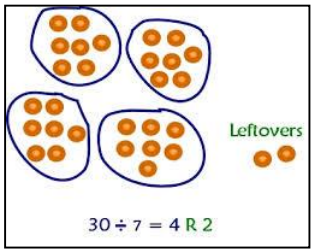
Misconceptions

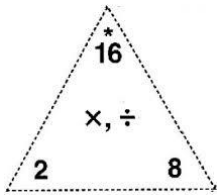
- Lack of understanding of ‘remainders’ and their importance to the context of the problem
- Insecure understanding of place value to know what each digit is representing
- Unable to derive facts from known facts and ‘play’ with numbers
- Approximations are wildly inaccurate so answers cannot be judged in the context of the problem/calculation
- No method to ‘fall back’ on where use of a formal method won’t work
- Instant recall of and strategies to quickly work out division facts related to the times tables for their year group should be taught.

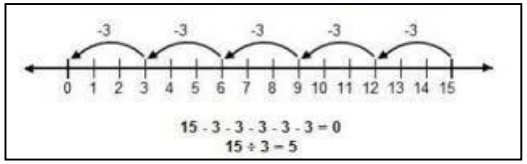
Models & Images

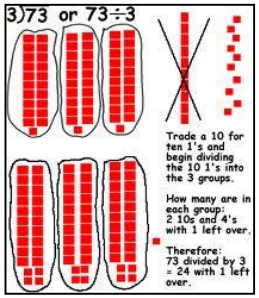












Linked Vocabulary

Divisor
Divisible
Divide
Group
Share
Chunk
Remainder
Sharing / shared
Equal groups